Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT3 **Branch: B.Tech (All)**

Semester : 2 **Date : 25/04/2018** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The infinite series $1+r+r^2+.....+r^{n-1}+\ldots$ is convergent if

(A) $|r|<1$ (B) $|r|>1$ (C) $r=1$ (D) $r<-1$

b) The sum of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....$ is

(A) $\log 2$ (B) zero (C) infinite (D) none of these

c) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^7 x\, dx$ is

(A) $\frac{32\pi}{35}$ (B) $\frac{32}{35}$ (C) zero (D) $\frac{16}{35}$

d) If $f_n=\int_0^{\frac{\pi}{4}} \tan^n x\, dx$, then $\left(f_n+f_{n-2}\right)$ is equal to ______.

(A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$ (C) $\frac{n}{n-1}$ (D) $\frac{n-1}{n}$

e) $\int_1^{\infty}\frac{1}{x^{\sqrt{2}}}\,dx$ is convergent.

(A) True (B) False

f) $\lceil n \lceil n-1 =$ _____

(A) $\frac{\pi}{\cos n\pi}$ (B) $\frac{\pi}{\sec n\pi}$ (C) $\frac{\pi}{\cos ec n\pi}$ (D) $\frac{\pi}{\sin n\pi}$

g) If $B(x,2)=\frac{1}{3}$, then the value of x = _____.

(A) 0 (B) 1 (C) 2 (D) none of these

h) If the power of y are even, then the curve is symmetrical about

(A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these

i) $\int_0^1 dx \int_0^x e^{\frac{y}{x}}\, dy$ is equal to

(A) $e+1$ (B) $e-1$ (C) $\frac{1}{2}(e+1)$ (D) $\frac{1}{2}(e-1)$

j) On converting into polar coordinates $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} dx\, dy$ is equal to

(A) $\int_0^{\pi}\int_0^{2a\cos\theta} r\, dr\, d\theta$ (B) $\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta} r\, dr\, d\theta$ (C) $\int_0^{\frac{\pi}{2}}\int_0^{2a\sin\theta} r\, dr\, d\theta$ (D) none of these

k) The transformations $x+y=u, y=uv$ transform the area element $dy\, dx$ into $|J|du\, dv$, where $|J|$ is equal to

(A) 1 (B) u (C) – 1 (D) none of these

l) The degree of the differential equation $3\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}$ is

(A) 1 (B) 2 (C) 3 (D) 6

m) The solution of $\frac{dy}{dx}=e^{x+y}$ is

(A) $e^x-e^{-y}=c$ (B) $e^x-e^y=c$ (C) $e^x+e^{-y}=c$ (D) $e^x+e^y=c$

n) The orthogonal trajectories of the family of curve $y=cx^k$ are given by

(A) $x^2+ky^2=$ const. (B) $x^2+cy^2=$ const. (C) $kx^2+y^2=$ const.

(D) $x^2-ky^2=$ const.

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Test the convergence of the series $\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+.....$ **(5)**

b) Using reduction formula evaluate: $\int_0^1 x^6\sin^{-1}x\, dx$ **(5)**

c) Prove that $\int_0^{\infty}\frac{x^4\left(1+x^5\right)}{(1+x)^{15}}\, dx=\frac{1}{5005}$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Prove that $\int_0^1 x^{q-1}\left(\log\frac{1}{x}\right)^{p-1} dx=\frac{\overline{|p}}{q^p}$. **(5)**

b) Using reduction formula prove that $\int_0^a x^5\left(2a^2-x^2\right)^{-3} dx=\frac{1}{2}\left(\log 2-\frac{1}{2}\right)$. **(5)**

c) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{\tan^{-1}n}{n^2+1}$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Change the order of integration in the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$ and evaluate it. **(5)**

b) Examine the series $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+.....+\frac{x^n}{n^2+1}+.....$ for convergence using ratio test. **(5)**

c) Solve: $\left(y^2 e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Solve: $\frac{dy}{dx}=2y\tan x+y^2\tan^2 x$ **(5)**

b) By changing into polar co-ordinates, evaluate the integral **(5)**

$\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} \left(x^2+y^2\right)\, dx\, dy$.

c) Using reduction formula, evaluate: $\int_0^\infty \frac{x^4}{\left(1+x^2\right)^4}\, dx$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Evaluate: $\int_0^\infty x^4 e^{-x^4} dx$ **(5)**

b) Solve: $xdy-ydx=\sqrt{x^2+y^2}\,dx$ **(5)**

c) Evaluate: $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dx\, dy\, dz}{\sqrt{1-x^2-y^2-z^2}}$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Trace the curve $r^2=a^2\cos 2\theta$. **(5)**

b) Evaluate: $\int_2^\infty \frac{x+3}{(x-1)\left(x^2+1\right)}\, dx$ **(5)**

c) Find the length of the arc of the curve $y=\log\sec x$ from $x=0$ to $x=\frac{\pi}{3}$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Show that the volume of the spindle-shaped solid generated by revolving the astroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ about the x-axis is $\frac{32\pi a^3}{105}$. **(5)**

b) Trace the curve $y^2(2a-x)=x^3$. **(5)**

c) Investigate the convergence of $\int_2^5 \frac{1}{\sqrt{(x-2)}}\, dx$. **(4)**